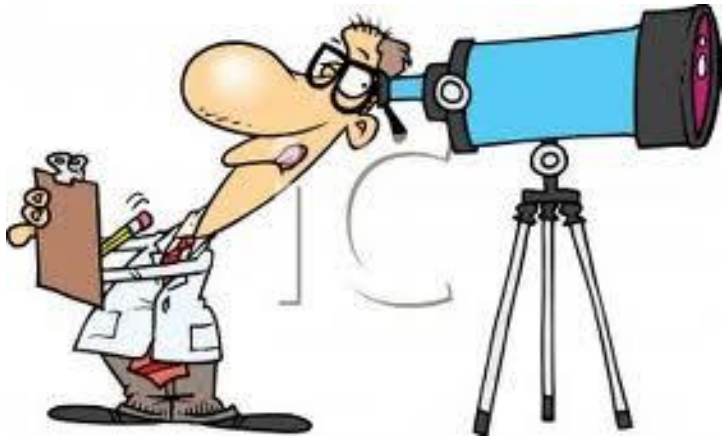# Number Bases

2/23/2013
Hui Ma

Using the most powerful telescope ever made, scientists observe a class of young aliens on a planet millions of light years away. On the blackboard, their teacher has written these equations:
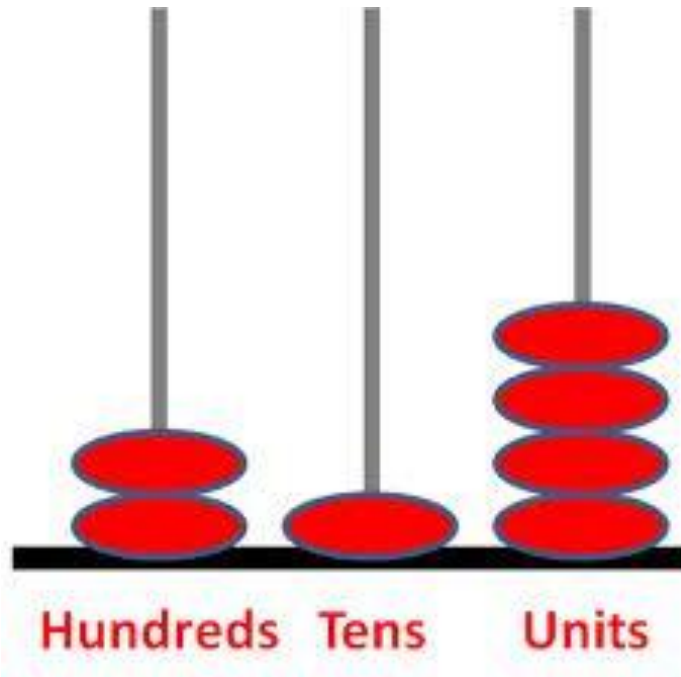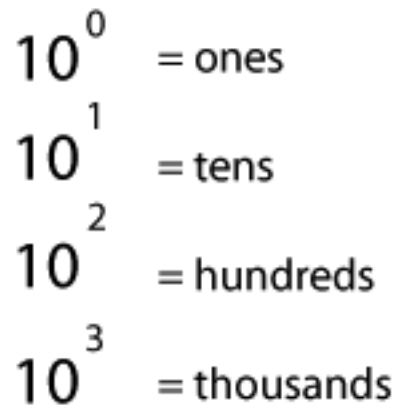
$$13 + 16 = 31$$
$$7 \cdot 3 = 25$$

How many fingers do they have?

How do we count?

DECIMAL SYSTEM (BASE 10 SYSTEM)
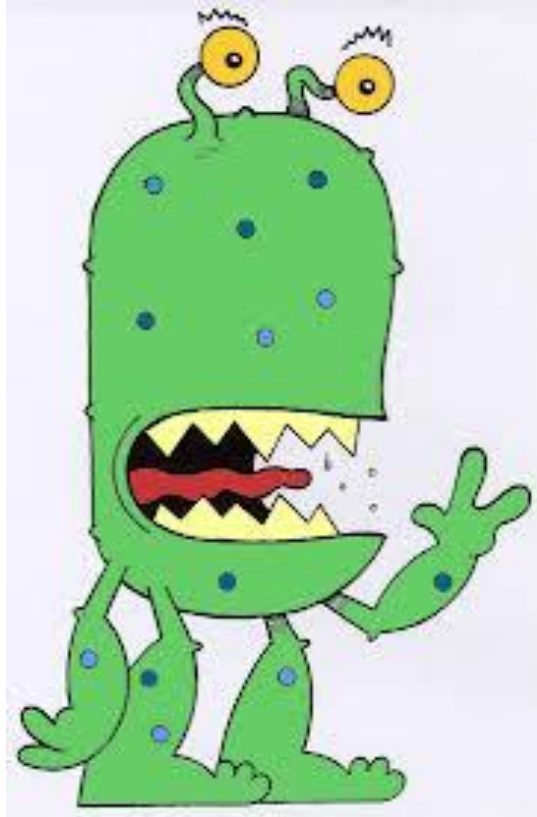
Using 10 digit symbols: 0, 1, 2, 3, …, 9

2 5 3 7

Thousands | Hundreds | Tens | Ones

10^0 = ones

10^1 = tens

10^2 = hundreds

10^3 = thousands

$2*1{,}000 + 5*100 + 3*10 + 7*1 = 2{,}537$

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CARTOON WORLD? ALIEN WORLD?

BASE 2 NUMBER SYSTEM (BINARY SYSTEM):

Using 2 digit symbols: 0, 1

Position:

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
128	64	32	16	8	4	2	1

Number:

1	0	1	1	0	0	0	1

$$(10110001)_2 = (177)_{10}$$

BASE 8 NUMBER SYSTEM (OCTAL SYSTEM):

Using 8 digit symbols: 0, 1, 2, 3, 4, 5, 6, 7

$$7 \quad 1 \quad 2 \quad 6 \quad 3$$
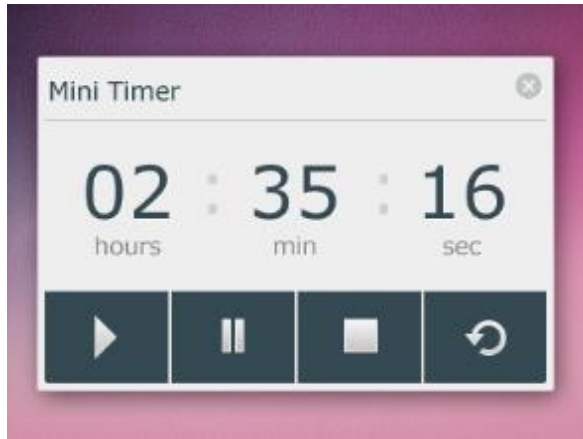$$8^4 \quad 8^3 \quad 8^2 \quad 8^1 \quad 8^0$$

decimal:

$$3 \times 8^0 = 3$$
$$6 \times 8^1 = 48$$
$$2 \times 8^2 = 128$$
$$1 \times 8^3 = 512$$
$$7 \times 8^4 = 28672$$
$$29363$$
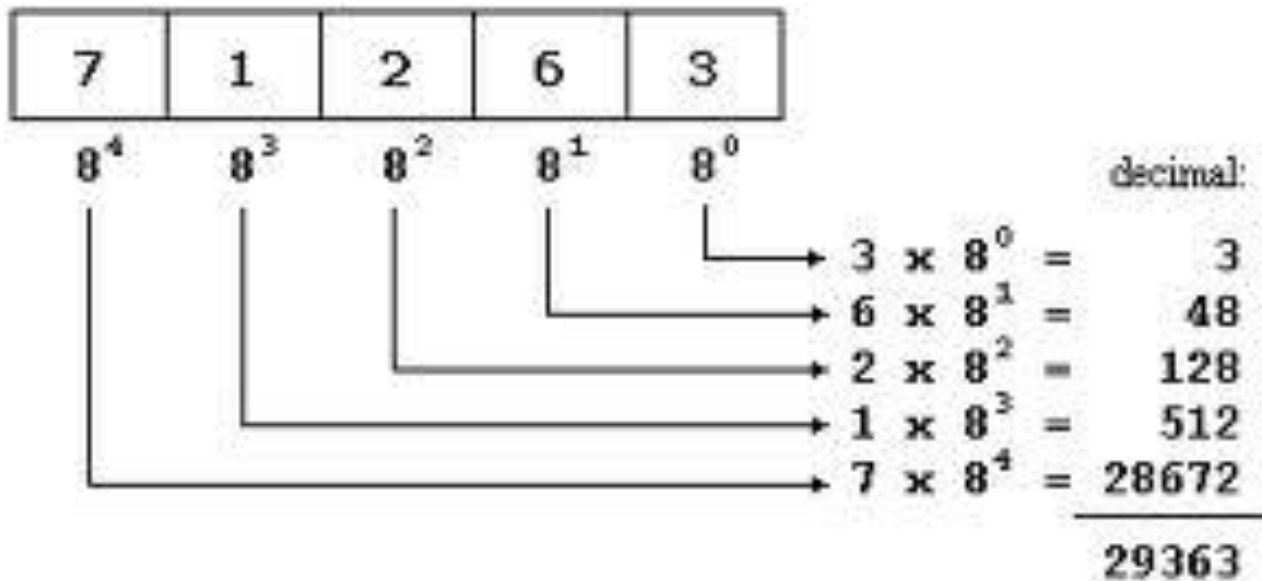
$$(71263)_2 = (29363)_{10}$$

BASE N NUMBER SYSTEM

Using n digit symbols: 0, 1,2,…, n-1

$$(a_k a_{k-1} \cdots a_2\, a_1 a_0)_n$$
$$= a_k n^k + a_{k-1} n^{k-1} + \cdots a_2\, n^2 + a_1 n^1 + a_0 n^0$$

where each a_i takes values from 0 to n-1
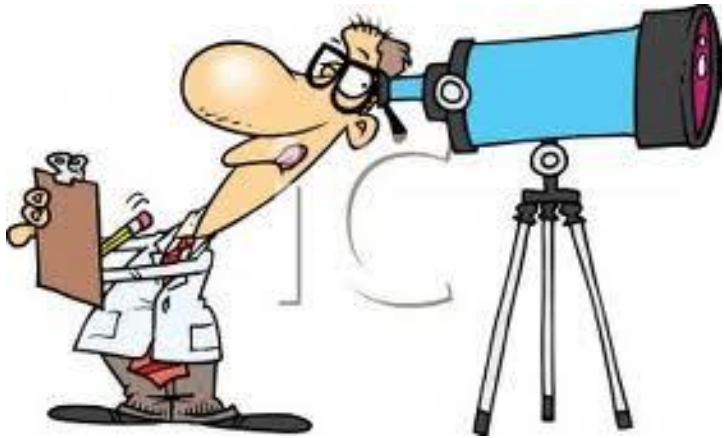
DIFFERENT BASES

Feet and inches: 3'5"

Pounds and ounces: 8lbs, 5 oz

9

CONVERTING DECIMAL TO OTHER BASES

Write the number 100 in the system with base 2 and 8.

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$$7 \cdot 3 = 25$$

How many fingers do they have?

ADD/MULTIPLY

A carry occurs each time the result of adding up digits in a column exceeds or equals the base!

$1100_2 + 1101_2$

$201_3 \times 102_3$

Using the most powerful telescope ever made, scientists observe a class of young aliens on a planet millions of light years away. On the blackboard, their teacher has written these equations:

$$13 + 16 = 31$$

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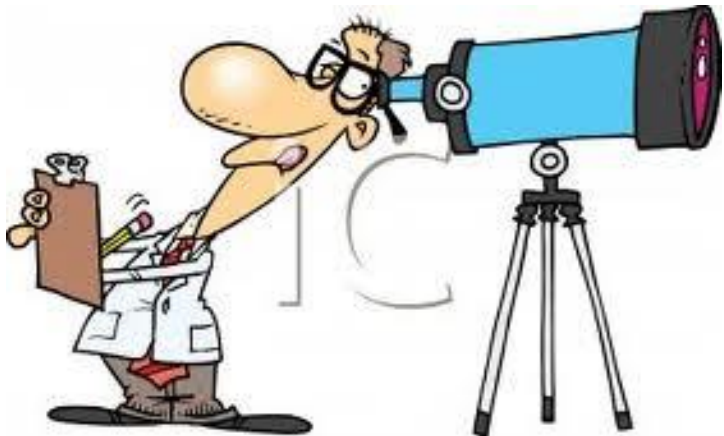
How many fingers do they have?

Binary	Octal	Decimal
0	0	0
1	1	1
10	2	2
11	3	3
100	4	4
101	5	5
110	6	6
111	7	7
1000	10	8
1001	11	9
1010	12	10