

Math Circle - Number bases
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1. In base 4 number system, you count as follows: 1, 2, 3, 10, 11, 12... What would the twentieth number be?
2. Convert the following numbers to base 10
 - 1) 1232_4
 - 2) 11.011_2
3. Convert the number 98 into
 - 1) base 7
 - 2) base 4
4. Add/multiply
 - 1) $21012_3 + 121201_3$
 - 2) $2111_3 \cdot 122_3$
5. In a system whose base is greater than 10, we need more than ten digit symbols, so we commonly use the letters A, B, C, ... for 10, 11, 12, ...etc. For example, in the base 11 system, 21_{10} could be written as 1A. Using this conversion, write the number 111_{10} in the base 11 notation.
6. Is it possible that the following statement are true in some number base system:
If so, what base?
 - 1) $3 \cdot 4 = 10$
 - 2) Both $3 + 4 = 10$ and $3 \cdot 4 = 15$
 - 3) Both $2 + 3 = 5$ and $2 \cdot 3 = 11$
7. An evil king wrote three secret two-digit numbers a, b, c. A handsome prince must name three numbers X, Y, Z, after which the king will tell him the sum $aX + bY + cZ$. The prince must then name all three of the King's numbers, or he will be executed. Help out the prince!

8. In the multiplication problem below, A, B, C, D are different digits. What is A+B?

$$\begin{array}{r} ABA \\ \times CD \\ \hline CD CD \end{array}$$

9. Solve for x and y: $51_x + 71_y = 10^2$

10. A three digit number \overline{xyz} in base 7 when written in base 9 becomes \overline{zyx} . What is the decimal representation of the number?

11. State and prove a condition (involving the representation of a number) which allows us to determine whether a number is even or odd:

- 1) In the base 3 system
- 2) In the base n system

12. Suppose I have four boxes with numbers (between 1 and 15) inside them.

A: 1, 3, 5, 7, 9, 11, 13, 15

B: 2, 3, 6, 7, 10, 11, 14, 15

C: 4, 5, 6, 7, 12, 13, 14, 15

D: 8, 9, 10, 11, 12, 13, 14, 15

If you pick a number between 1 and 15 and tell me all of the boxes it is in, I can tell you the number (try this if you want!). How do I do it?