Graph coloring problem set

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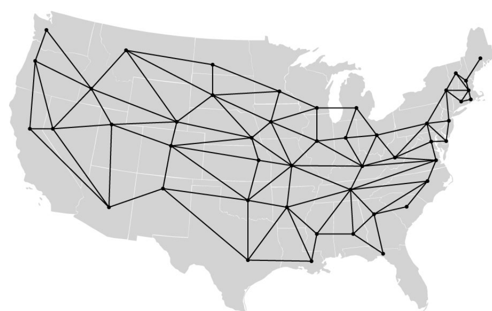
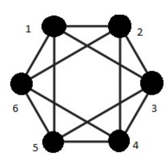
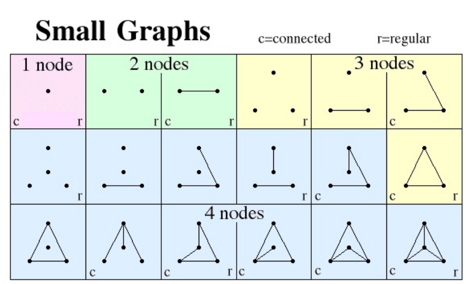
1. How many colors are required to color the following graph? (We know it can be done with 4. Can it be done with 3?)



2. How many colors are required to color the following graph? (That is, what is the graph's chromatic number?) (Hint: we said in the lecture that we *can't* do this with 3 colors. Can we do it with 4?)



3. What are the chromatic numbers of the following graphs? (This seems like a lot, but many of these will be pretty simple. Hint: the first one may be properly colored with one color, but not with zero colors, so its chromatic number is 1.)



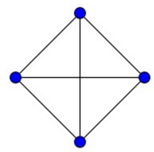
(From Wikipedia)

4. Suppose you arrange 15 billiard balls in a triangular array, like for a game of 8-ball. (That is, a row of 1 ball, then a row of 2 balls, then 3, then 4 then 5.) How many colors would you need so that no ball touches a ball of the same color?

5. Here's a picture of the Dodge Pentastar logo, from the bumper of a 1998 Jeep Cherokee (photo taken and uploaded to Wikipedia by Christopher Ziemnowicz):

Consider this shape as a graph with 6 vertices (the 5 corners plus the middle) and 10 edges (the 5 sides outside plus 5 edges to the middle vertex). What is that graph's chromatic number?

6. Is the following graph considered "planar"? That is, could it be redrawn so that its edges do not cross? (Can you move the vertices around, and/or curve/stretch the edges, so that the edges don't cross?) Also, what is its chromatic number? FYI, this graph is called the "Complete Graph on 4 Vertices", because it has 4 vertices and every vertex is adjacent to every other vertex.



7. Which of the graphs in Questions 1-5 are planar? Note: the hardest one is the graph in question 2. (For question 2, this problem is asking: can you place 6 vertices on a page and make edges connecting vertices 1-2, 1-3, 1-5, 1-6, 2-3, 2-4, 2-6, 3-4, 3-5, 4-5, 4-6, 5-6, that don't cross? The edges are allowed to curve.)

8. (From Tucker, *Applied Combinatorics*) A set of solar experiments is to be made at observatories. Each experiment begins on a certain day of the year and ends on a certain day, and will be repeated each year. Each observatory can only run one experiment at a time. What is the minimum number of observatories necessary to perform the experiments? Draw a graph with vertices and edges to turn this into a graph coloring problem, then solve it (find the graph's chromatic number).

   Experiment A runs from Sep 2 to Jan 3 each year

   Experiment B: Oct 15 to Mar 10

   C: Nov 20 to Feb 17

   D: Jan 23 to May 30

   E: Apr 4 to Jul 28

   F: Apr 30 to Jul 28

   G: Jun 24 to Sep 30

9. An *edge coloring* of a graph assigns colors to the edges of a graph, so that edges with a common end vertex receive different colors. Describe graphs that can be edge colored using just two colors.

10. In a "round-robin" tournament where every team plays each other, a major problem is scheduling the play over the least number of days possible. (Suppose each team plays at most one match per day.)
    (a) Find the fewest number of days possible for a round-robin tournament if $n = 4$ (so there are 4 teams, and they all have to play each other).
    (b) Restate this problem as an *edge coloring* problem (see previous problem). (What do the vertices, edges and colors represent? Why can't one vertex be attached to two edges that have the same color?) Solve the problem for $n = 4$ and see if it agrees with your answer in part (a).
    (c) For $n = 5$ teams, I think this is harder to solve. However, here are some hints: Each day there can be at most 2 games played (at most 4 teams play and at least one team is idle each day). How many games must be played total? In that case, what is the least possible number of days it can take? Can you come up with a coloring that achieves this?