**Black Hills Math Circle**

**Topic: Mathematical Induction**

**Meeting Number:**3

**Date:11/17/2012**

**Let** $P_1, P_2, P_3, \ldots$ **be a list of propositions that may or may not be true. Principle of Mathematical Induction: It asserts of that all the statements** $P_1, P_2, P_3, \ldots$ **are true provided: (i) Basis Step: Show that** $P_1$ **is true. (ii) Inductive Step: Show that** $P_{n+1}$ **is true whenever** $P_n$ **is true.**

**Problem 1:**

*Is* $(11)^n - 4^n$ *where* $n$ *is a natural number, divisible by* $7$*?*

**Problem 2:**

*Is* $7^n - 6n - 1$ *where* $n$ *is a natural number, divisible by* $36$*?*

**Problem 3:**

*Prove or disprove that* $n^2 + n + 41$ *is a prime number* $n \geq 0$.

**Problem 4:**

*Prove that* $1 + 2 + 3 + \cdots + n = n(n+1)/2$.

**Problem 5:**

*Prove that* $1^2 + 2^2 + 3^2 + \cdots + n^2 = n(n+1)(2n+1)/6$.

**Problem 6:**

*Prove that* $1^2 + 3^2 + 5^2 + \cdots + (2n-1)^2 = (4n^3 - n)/3$.

**Problem 7:**

*Prove that* $(1 + 2 + 3 + \cdots + n)^2 = 1^3 + 2^3 + 3^3 + \cdots + n^3$.

**Problem 8:**

*Let* $n$ *be a natural number. Show that any* $2^n \times 2^n$ *chessboard with one square removed can be tiled using L-shaped pieces.*